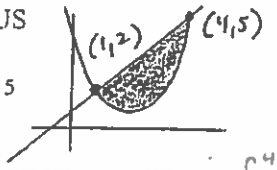
# SOLIDS OF KNOWN CROSS SECTION

Key

AP CALCULUS NAME

1. $y = x+1$, $y = x^2-4x+5$




a. Area

$\int_1^4 x+1-(x^2-4x+5)\,dx = \int_1^4 -x^2+5x-4\,dx = -\frac{1}{3}x^3+\frac{5}{2}x^2-4x\Big|_1^4$ $-\frac{64}{3}+40-16-(-\frac{1}{3}+\frac{5}{2}-4)$

$-\frac{63}{3}+24+1.5$

$3+1.5=4.5$

b. cross sections perpendicular to $x$ axis are squares

$A=s^2$ $s = x+1-(x^2-4x+5)$ $\int_1^4 (-x^2+5x-4)^2\,dx$

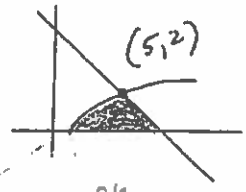
c. cross sections perpendicular to $x$ axis are rectangles with height of 3

$A=3\cdot s$ $\int_1^4 -3x^2+15x-12\,dx$

d. cross sections perpendicular to $x$ axis are semicircles

$A=\frac{1}{2}\pi r^2$ $\int_1^4 \frac{1}{2}\pi(-x^2+5x-4)^2$

2. $y=\sqrt{x-1}$, $y=12-2x$




a. Area

$\int_1^5 \sqrt{x-1}\,dx + \int_5^6 12-2x = \frac{2}{3}\sqrt{x-1}\Big|_1^5 + 12x-x^2\Big|_5^6 = \frac{4}{3}-0+(36-35)=\frac{4}{3}+1=\frac{7}{3}$

b. cross sections perpendicular to $x$ axis are squares

$A=s^2$ $\int_1^5 (\sqrt{x-1})^2 + \int_5^6 (12-2x)^2\,dx = \frac{1}{2}x^2-x\Big|_1^5 + 144x-2x^2+\frac{4}{3}x^3\Big|_5^6 = 7.5-0.5+(1080-970)$

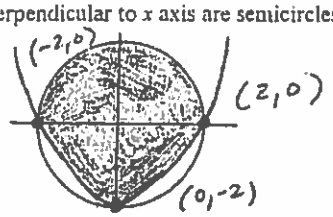
c. cross sections perpendicular to $x$ axis are rectangles with height of 2

$A=2\ell$ $V=\int_1^5 2(\sqrt{x-1})\,dx + \int_5^6 2(12-2x)\,dx$

d. cross sections perpendicular to $x$ axis are semicircles

$A=\frac{1}{2}\pi(\frac{d}{2})^2$ $V=\int_1^5 \frac{\pi}{2}\left(\frac{\sqrt{x-1}}{2}\right)^2 dx + \int_5^6 \frac{\pi}{2}\left(\frac{12-2x}{2}\right)^2 dx$

3. $y=\sqrt{4-x^2}$, $x^2+y^2=4$, $y=\frac{1}{2}x^2-2$




a. Area

$A=\int_{-2}^{2} \sqrt{4-x^2}-(\frac{1}{2}x^2-2)\,dx = 11.617$

b. cross sections perpendicular to $x$ axis are squares

$A=\ell^2$ $V=\int_{-2}^{2}\left[\sqrt{4-x^2}-(\frac{1}{2}x^2-2)\right]^2 dx = 38.050$

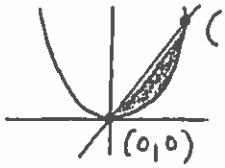
c. cross sections perpendicular to $x$ axis are rectangles with height of 5

$A=5\ell$ $V=\int_{-2}^{2} 5\left(\sqrt{4-x^2}-(\frac{1}{2}x^2-2)\right)dx = 58.083$

d. cross sections perpendicular to $x$ axis are semicircles

$A=\frac{1}{2}\pi(\frac{d}{2})^2$ $V=\int_{-2}^{2}\frac{\pi}{2}\left(\frac{\sqrt{4-x^2}-(\frac{1}{2}x^2-2)}{2}\right)^2 dx = 14.942$

4. $y=x^2$, $y=3x$




a. Area

$A=\int_0^3 3x-x^2\,dx = 4.5$

b. cross sections perpendicular to $y$ axis are squares

$A=s^2$ $V=\int_0^3 (3x-x^2)^2\,dx = 8.1$

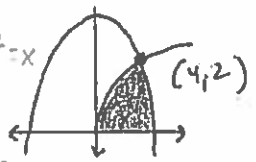
c. cross sections perpendicular to $y$ axis are rectangles with height of 4

$A=4\ell$ $V=\int_0^3 4(3x-x^2)\,dx = 18$

d. cross sections perpendicular to $y$ axis are semicircles

$A=\frac{\pi}{2}(\frac{d}{2})^2$ $V=\int_0^3 \frac{\pi}{2}\left(\frac{3x-x^2}{2}\right)^2 dx = 3.18$

5. $y=\sqrt{x}$ $y^2=x$
$y=-\frac{1}{4}x^2+6$




$\sqrt{24-4y}=x$

a. Area
$A=\int_0^2 \sqrt{24-4y}-y^2\,dy = 6.263$

b. cross sections perpendicular to $y$ axis are squares
$A=s^2$ $\int_0^2 (\sqrt{24-4y}-y^2)^2\,dy = 23.793$

c. cross sections perpendicular to $x$ axis are squares
$A=s^2$ $V=\int_0^4 (\sqrt{x})^2dx+\int_4^{\sqrt{24}} (-\frac{1}{4}x^2+6)^2dx = 8+1.260 = 9.260$

d. cross sections perpendicular to $y$ axis are rectangles with height of 1
$A=1\cdot l$ $V=\int_0^2 1\cdot(\sqrt{24-4y}-y^2)\,dy = 6.263$

e. cross sections perpendicular to $x$ axis are semicircles.
$A=\frac{1}{2}\pi(\frac{d}{2})^2$ $V=\int_0^4 \frac{\pi}{2}\left(\frac{\sqrt{x}}{2}\right)^2dx+\int_4^{\sqrt{24}} \frac{\pi}{2}\left(\frac{-\frac{1}{4}x^2+6}{2}\right)^2dx = 3.142+0.495$
$=3.637$

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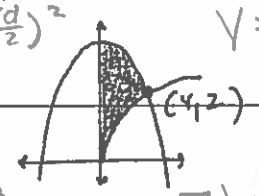
6. $y=\sqrt{x}$
$y=-\frac{1}{4}x^2+6$

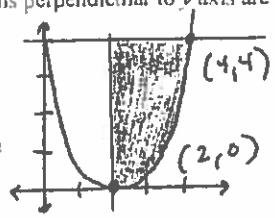



a. Area
$A=\int_0^4 (-\frac{1}{4}x^2+6-\sqrt{x})\,dx = 13.333$

b. cross sections perpendicular to $x$ axis are squares
$A=s^2$ $V=\int_0^4 (-\frac{1}{4}x^2+6-\sqrt{x})^2dx = 55.086$

c. cross sections perpendicular to $y$ axis are squares
$A=s^2$ $V=\int_0^2 (y^2)^2dy+\int_2^6 (\sqrt{24-4y})^2dy = 6.4+32$
$=38.4$

7. $x=2$
$y=4$
$y=(x-2)^2$




$\sqrt{y}+2=x$

a. Area
$A=\int_2^4 4-(x-2)^2\,dx = 5.333$

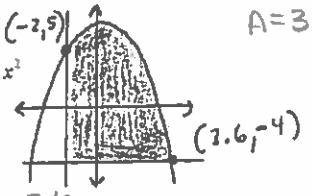
b. cross sections perpendicular to $x$ axis are squares
$A=s^2$ $V=\int_2^4 (4-(x-2)^2)^2dx = 17.067$

c. cross sections perpendicular to $y$ axis are squares
$A=s^2$ $V=\int_0^4 (\sqrt{y}+2-2)^2dy = 8$

d. cross sections perpendicular to $x$ axis are rectangles with height of 2
$A=2\cdot l$ $V=\int_2^4 2\,(4-(x-2)^2)\,dx = 10.667$

e. cross sections perpendicular to $y$ axis are rectangles with height of 3
$A=3\cdot l$ $V=\int_0^4 3\,(\sqrt{y}+2-2)\,dy = 16.000$

8. $y=9-x^2$
$x=-2$
$y=-4$




a. Area
$\int_{-2}^{3.6} 9-x^2-(-4)\,dx = 54.581$

b. cross sections perpendicular to $x$ axis are squares
$A=s^2$ $\int_{-2}^{3.6} (13-x^2)^2dx = 600.047$

c. cross sections perpendicular to $y$ axis are squares
$A=s^2$ $\int_{-4}^{9} (\sqrt{9-y})^2dy+\int_{-4}^{9} (-\sqrt{9-y}+2)^2dy = 96.008$
$84.5+11.508$