

OPTIMIZATION  
AP CALCULUS

Answers  
NAME \_\_\_\_\_

Calculator permitted. Round or truncate all decimals to 3 decimal places.

1. An open box is to be made from a square piece of material with an area of  $100 \text{ cm}^2$  by cutting equal squares from the corner and turning up the sides. Find the maximum volume of the box.

$$V = \frac{2000}{27} \text{ cm}^3$$

2. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least (minimum) amount of fencing if no fencing is needed along the river?

$$300 \times 600$$

3. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular regions. What dimensions should be used to enclose a maximum area?

$$\frac{100}{3} \times 25$$

4. A rectangular solid is to be made with a square base and a surface area of  $150 \text{ in}^2$ . Determine the maximum volume.

Don't Do

In numbers 5-6, find the point of the graph of the function that is closest to the given point. (When is the distance between them a minimum.)

5.  $f(x) = \sqrt{x-8}; (2,0)$

$$(8,0)$$

6.  $f(x) = x^2; \left(2, \frac{1}{2}\right)$

$$(1,1)$$

7.  $f(x) = (x+1)^2; (5,3)$

$$(1,4)$$

8. A rectangle is bounded by the x axis, y axis and the graph of  $y = \frac{6-x}{2}$ . Determine the maximum area of the rectangle.

$$A = \frac{9}{2}$$

9. A rectangle is bounded by the x axis, and the semicircle  $y = \sqrt{25-x^2}$ . What length and width should the rectangle have so that its area is a maximum?

$$5\sqrt{2} \times \frac{5\sqrt{2}}{2}$$

10. A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.

$$\frac{32}{4+\pi} \times \frac{16}{4+\pi}$$

11. A particle moves along the x axis such that its distance from the origin is represented by  $x(t) = t^3 - \frac{3}{2}t^2 - 6t$  where  $0 \leq t \leq 6$ .

a) What is the particles maximum distance from the origin?

$$126$$

b) What is the particles maximum velocity?

$$84$$

c) What is the particles minimum velocity?

$$-6.75$$

12. A particle moves along the x axis such that its distance from the origin is represented by  $x(t) = t^3 - 6t^2 + 9t - 1$  where  $0 \leq t \leq 5$ .

a) What is the particles maximum distance from the origin?

$$19$$

b) What is the particles maximum velocity?

$$24$$

c) What is the particles minimum velocity?

$$-3$$

13. A person in a rowboat two miles from the nearest point on a straight shoreline wishes to reach a house six miles farther down the shore. If the person can row at a rate of 3 mph and walk at a rate of 5 mph, find the least amount of time required to reach the house. How far from the house should the person land the rowboat?

$$4.5 \text{ miles}$$

$$\frac{26}{15} \text{ hours} / 1.733 \text{ hours}$$